

# The Solution of Tachyon Inflation in Curved Universe

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**Abstract** In this paper, we have considered the curved universe which is filled by tachyonic field. We have found the exact solutions for the field, pressure, density, and scale factor and some cosmological parameters. In such universe, we have investigated the role of tachyonic field in different stages of  $k$  for the evolution of the universe. Finally we draw the graphs for the scale factor, Hubble's parameter, energy density, pressure, acceleration parameter, equation of state and potential for the different values of  $k$ . Also we obtained the exact form of field which shows that the tachyonic field has the kink form.

**Keywords** Tachyonic field · Curved universe · Acceleration parameter

## 1 Introduction

In recent decade, it is observed that the space of the universe increases with an acceleration rate. Expansion of the universe arise from energy called dark energy, and it is almost three-quarters of the total mass-energy of the universe. As we know there are few more taxing questions facing cosmology today than what is nature of the dark energy in the universe? Over the past few years there have been many papers devoted in addressing the nature of the dark energy and accelerated expansion of universe. On the other hand there have been difficulties in obtaining accelerated expansion from fundamental theories such as string theory [1–4]. Much has been written and emphasized about the role of the fundamental dilation field in the context of string cosmology, but not much emphasis on tachyon component.

In this paper, we turn our attention to the issue of the tachyon as a source of the dark energy. As we know the tachyon is an unstable field which has become important in string

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theory through its role in the Dirac-Born-Infeld (DBI) action which is used to describe the  $D$ -brane action [5–11]. A number of authors have already demonstrated that the tachyon could play an important role in cosmology [12–14], independent of the fact that it is an unstable field. It can act as a source of dark matter and can lead to a period of inflation depending on the form of the associated potential. Indeed it has been proposed as the source of dark energy for a particular class of potential [15–17]. However, there has not really been an effect to understand the general properties of tachyonic cosmologies. In order to attempt the problem, we start four-dimensional DBI action where the tachyon field is coupled to a background of perfect fluid with radiation or matter.

Also, we consider model of Friedmann-Robertson-Walker (FRW) cosmology with curvature, driven by real scalar field which evolves with standard or tachyonic dynamics. We note that in the Ref. [18], they have assumed that the universe is filled in tachyonic field with potential and discussed the acceleration of the universe. In that case they have considered the FRW model with metric as a spatially flat. We have assumed that the universe is filled in only tachyonic field but the FRW metric is curved. So, for tachyonic dynamics we extend the result of [18], which was in flat space-time. In contrast to the paper assuming the tachyon potential, we have also obtained the explicit form of potential for the tachyon field. This paper is organized as follows: Sect. 2 we study tachyon dynamic with FRW standard model in curved universe. With the help of energy-momentum tensor we obtain the equation of motion for the corresponding tachyonic field. The solution for the tachyonic equation is presented in Sect. 3. In this section we obtain the exact solution for some cosmological parameters in curved universe and show the accelerating expansion of our universe due to tachyonic field. Finally, in Sect. 4 offer some closing remarks and results.

## 2 Tachyon Dynamic

One of considerable methods inflationary describe potential or vacuum energy of scalar field with tachyon field. We can see details of tachyon field in Refs. [19–21]. However, we want to investigate a cosmological scenario to help tachyon field  $T$  in a perfect fluid. Now we begin the single tachyonic inflation model with applying Lagrange density in the DBI type action [22–24]. So, the action for the homogenous tachyon condensate of string theory in gravitational background is given by,

$$S = \int d^4x \sqrt{-g} \left( \frac{M_p^2}{2} R + \mathcal{L} \right), \tag{1}$$

where  $M_p = (8\pi G)^{-1/2}$  is reduced Plank’s mass when we take  $M_p = 1/\sqrt{2}$ . Also  $R$  and  $\mathcal{L}$  are scalar curvature and Lagrangian density respectively.

The Lagrangian density  $\mathcal{L}$  is

$$\mathcal{L} = -V(T)\sqrt{1 - \partial_\mu T \partial^\mu T}, \tag{2}$$

where  $V(T)$  is tachyonic potential. We use spatially metric in FRW standard model as,

$$ds^2 = -dt^2 + a^2(t) \left( \frac{dr^2}{1 - k r^2} + r^2 d\Omega^2 \right), \tag{3}$$

where constant of  $k = 1, 0$  and  $-1$  is for spherical, flat and hyperbolic geometry respectively. We obtain Ricci tensor,  $R_{\mu\nu}$ , and Ricci scalar,  $R$ , as functional of scale factor,  $a(t)$ , by,

$$R_{00} = -3\frac{\ddot{a}}{a}, \quad (4)$$

$$R_{ij} = g_{ij} \left( \frac{\ddot{a}}{a} + 2\frac{\dot{a}^2}{a^2} + 2\frac{k}{a^2} \right), \quad i, j = 1, 2, 3, \quad (5)$$

$$\alpha^2 = 2 \left( \frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} \right), \quad \text{that } \alpha = \sqrt{R/3}. \quad (6)$$

The energy momentum tensor for the tachyonic field is

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}} = -V(T) \sqrt{1 + g^{\mu\nu} \partial_\mu T \partial_\nu T} g^{\mu\nu} + \frac{V(T) \partial_\mu T \partial_\nu T}{\sqrt{1 + g^{\mu\nu} \partial_\mu T \partial_\nu T}}. \quad (7)$$

In FRW metric with a scale factor  $a$ , the pressure and energy densities of the field  $T$  are given respectively,

$$\rho = \frac{V(T)}{\sqrt{1 - \dot{T}^2}}, \quad p = -V(T) \sqrt{1 - \dot{T}^2}. \quad (8)$$

In order to obtain the corresponding tachyonic field equation we use energy-momentum tensor as  $T_\nu^\mu = (-\rho, p, p, p)$  and Einstein's equations as  $G_\nu^\mu = 2 T_\nu^\mu$ , so we have,

$$H^2 = \left( \frac{\dot{a}}{a} \right)^2 = \frac{2}{3} \rho - \frac{k}{a^2} = \frac{2}{3} \frac{V}{\sqrt{1 - \dot{T}^2}} - \frac{k}{a^2}, \quad (9)$$

$$\dot{H} = -(p + \rho) + \frac{k}{a^2} = -\frac{V \dot{T}^2}{\sqrt{1 - \dot{T}^2}} + \frac{k}{a^2}, \quad (10)$$

where  $H$  is Hubble's parameter.

We rewrite the equation of motion as,

$$\frac{\ddot{T}}{1 - \dot{T}^2} + 3H\dot{T} + \frac{1}{V} \frac{dV}{dT} = 0. \quad (11)$$

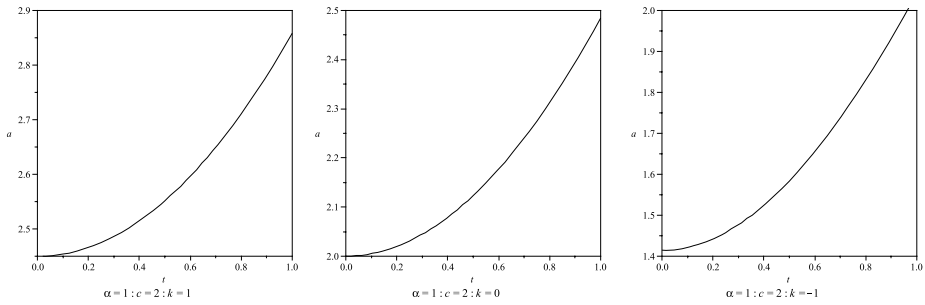
We note that the tachyonic potential and field in terms of scale factor can be written by the following respectively,

$$V = \frac{\sqrt{3}}{2} \sqrt{\frac{\alpha^4}{4} - \frac{\dot{a}^2}{a^2}}, \quad (12)$$

$$\dot{T}^2 = \frac{2}{3} \left( 1 + \frac{1}{1 - \frac{\alpha^2}{2} \frac{a}{\dot{a}}} \right). \quad (13)$$

As we know, the acceleration parameter and equation state are respectively,

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -1 - \frac{\dot{H}}{H^2}, \quad (14)$$



**Fig. 1** Graphs of the scale factor

$$\omega = \frac{p}{\rho}, \tag{15}$$

so we have,

$$\omega = \dot{T}^2 - 1, \tag{16}$$

$\omega$  is a number depending on fluid types, in that case  $\omega = 0$  for dust and  $\omega = 1/3$  for radiation. Also condition of universe expansion is  $\omega < -1/3$ , as it express an unknown energy that is called dark energy.

### 3 Solution for the Curved Universe

By using (6) and choosing  $\alpha = Constant$ , the scale factor can be written by,

$$a(t) = \pm \frac{1}{\alpha} \sqrt{2k - \alpha(c_1 e^{\alpha t} - c_2 e^{-\alpha t})}, \tag{17}$$

where  $k$  is curved geometry,  $c_1, c_2$  are integration constants and  $\alpha$  is functional of Ricci scalar ( $R$ ). For simplicity, here we take  $c_2 = -c_1 = c$  and rewrite scale factor as,

$$a(t) = \pm \frac{\sqrt{2}}{\alpha} \sqrt{k + \alpha c \cosh(\alpha t)}. \tag{18}$$

Now we draw the variation of scale factor with respect to time and we have Fig. 1. It show that scale factor increase as a positive, so the universe expands eternally.

The Hubble’s parameter is obtained by (9) as follows,

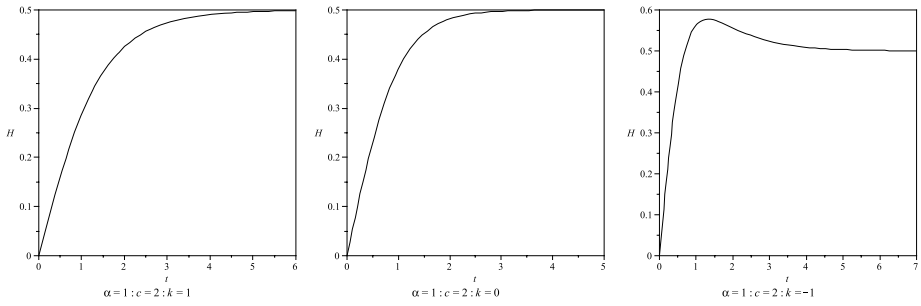
$$H(t) = \frac{1}{2} \frac{\alpha^2 c \sinh(\alpha t)}{k + \alpha c \cosh(\alpha t)}. \tag{19}$$

Also we draw the Hubble’s parameter in term of time, see Fig. 2. The Hubble’s parameter with the choice of different values of  $k$ , asymptotically increase with constant value.

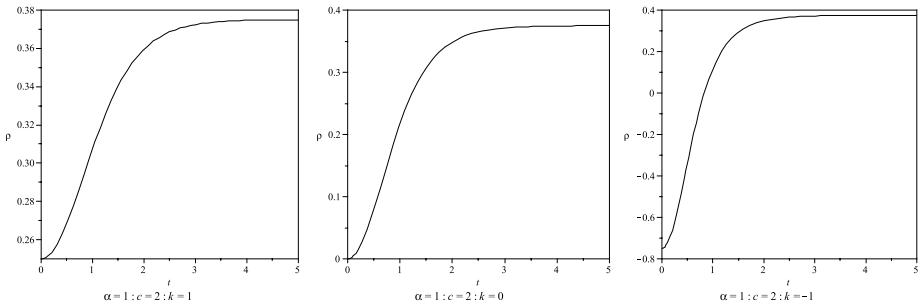
The energy density and pressure are obtained as,

$$\rho(t) = \frac{3\alpha^2}{8} \frac{\alpha^2 c^2 \cosh^2(\alpha t) + 2k\alpha c \cosh(\alpha t) + 2k^2 - \alpha^2 c^2}{(k + \alpha c \cosh(\alpha t))^2}, \tag{20}$$

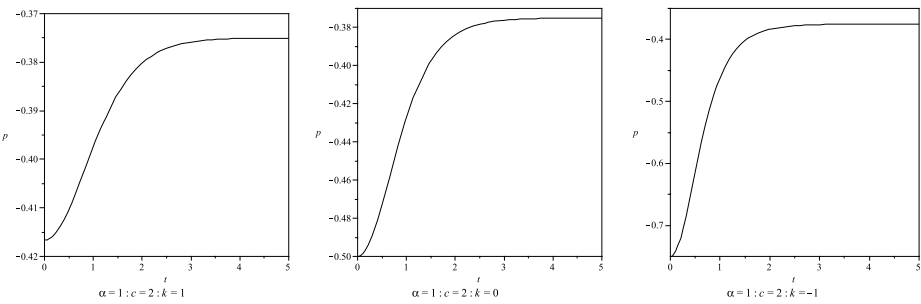
$$p(t) = -\frac{\alpha^2}{8} \frac{3\alpha^2 c^2 \cosh^2(\alpha t) + 6k\alpha c \cosh(\alpha t) + 2k^2 + \alpha^2 c^2}{(k + \alpha c \cosh(\alpha t))^2}. \tag{21}$$



**Fig. 2** Graphs of the Hubble’s parameter



**Fig. 3** Graphs of the energy density



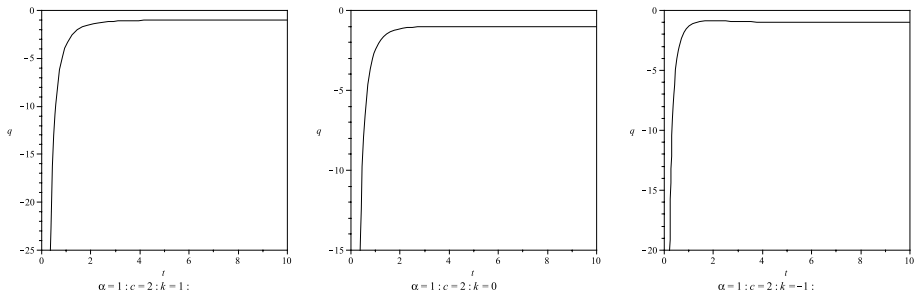
**Fig. 4** Graphs of the pressure

The variation of energy density with respect to time is plotted in Fig. 3. The energy densities for  $k = 0$  and  $k = 1$  start to increase from positive value and tend to asymptotically increase to a value of positive constant.

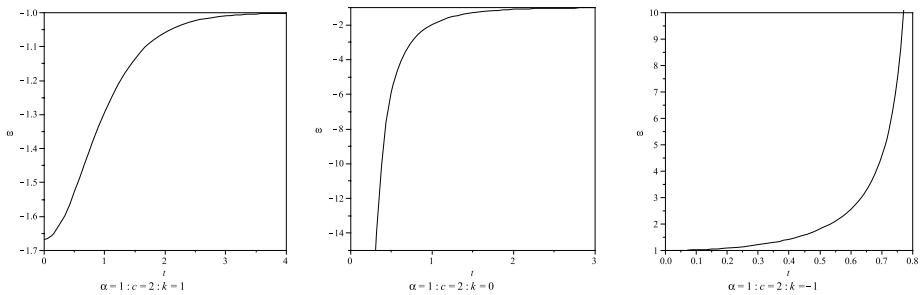
Also we note that in Fig. 4 for the different value of  $k$  the pressure start to increase asymptotically to negative value.

From (14) and (18) the acceleration parameter can be written as,

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = -\frac{c\alpha \cosh^2(\alpha t) + 2k \cosh(\alpha t) + c\alpha}{c\alpha \sinh^2(\alpha t)}. \tag{22}$$



**Fig. 5** Graphs of the acceleration parameter



**Fig. 6** Graphs of the equation of state

In Fig. 5, we see acceleration parameters increase from a negative value and tend to asymptotically approach  $-1$ .

By using (16) and (18) one can obtain the equation of state as a following,

$$\omega = -\frac{1}{3} \frac{3\alpha^2 c^2 \cosh^2(\alpha t) + 6k\alpha c \cosh(\alpha t) + 2k^2 + \alpha^2 c^2}{\alpha^2 c^2 \cosh^2(\alpha t) + 2k\alpha c \cosh(\alpha t) + 2k^2 - \alpha^2 c^2}. \tag{23}$$

We see that in Fig. 6 the equation of state for  $k = 0$  and  $k = 1$  start to increase and asymptotically approach to negative constant. But in case of  $k = -1$  the equation of state also increases from positive value and we see some singularity.

The corresponding potential for the tachyon field in terms of time can be obtain by (12) and (18),

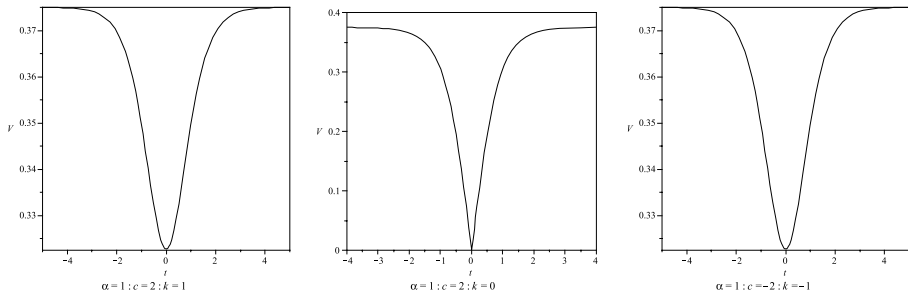
$$V = \frac{\sqrt{3}\alpha^2}{8} \sqrt{4 - \frac{\alpha^2 c^2 (\alpha c \cosh^2(\alpha t) + 2k \cosh(\alpha t) + \alpha c)^2}{(k + \alpha c \cosh(\alpha t))^4}}. \tag{24}$$

In order to obtain the tachyon field we substitute (17) in (13) and draw the following  $\dot{T}$  in terms of time, in that case this graph will be equivalent to  $\beta \operatorname{sech}(\alpha t)$ .

$$\dot{T} = \frac{2\sqrt{3}}{3} \frac{\sqrt{k^2 - \alpha^2 c^2}}{\sqrt{\alpha^2 c^2 \cosh^2(\alpha t) + 2k\alpha c \cosh(\alpha t) + 2k^2 - \alpha^2 c^2}} \tag{25}$$

and

$$\dot{T} = \frac{2\sqrt{3}}{3} \frac{\sqrt{k^2 - \alpha^2 c^2}}{\alpha c} \operatorname{sech}(\alpha t) = \beta \operatorname{sech}(\alpha t), \tag{26}$$



**Fig. 7** Graphs of the potential

so the tachyon field obtains as,

$$T = \frac{\beta}{\alpha} \arctan(\sinh(\alpha t)). \quad (27)$$

The behavior of this field in terms of time can be form of kink-like, which is important to tachyon condensation.

#### 4 Conclusion

In this letter, we have considered the curve FRW universe driven by only tachyonic field. We have presented accelerating expansion of our universe due to tachyonic field. We also found exact solution of tachyonic field which is given by (27). This field leads us to obtained the corresponding potential for the tachyon field. In this paper we assumed that  $\alpha = \text{constant}$  so the  $R$  curvature stays constant. The interesting problem here is to find solution of tachyon field for general  $f(R)$  instead of  $R$ . This problem will be investigated in the future.

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